

Controlling friction

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Two different controlling methods are proposed to stabilize unstable continuous-sliding states of a dry-friction oscillator. Both methods are based on a delayed-feedback mechanism well known for stabilizing periodic orbits in deterministic chaos. The feedback variable is the elastic deformation. The control parameter is either the sliding velocity or the normal force. We calculate analytically stability boundaries in the space of control parameter and delay time. Furthermore, we show that our methods are able to turn stick-slip motion into continuous sliding. Controlling friction helps to get a better understanding of friction by measuring, e.g., velocity-weakening friction forces. [S1063-651X(98)50305-4]

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If one tries to move two contacting solid bodies laterally, one often observes stick-slip motion due to dry friction (i.e., solid-solid friction with or without lubricants) [1]. This motion is characterized by a (more or less) periodic switching between sticking (relative sliding velocity is zero) and slipping (relative sliding velocity is on average much larger than the applied velocity). This stick-slip motion is responsible for the everyday experience of singing violins and squeaking doors. In most technological cases one wants to avoid stick-slip motion because it leads to vibrations and wear. The goal is to bring the system into the continuous sliding state, where the relative sliding velocity is constant and does not oscillate.

The classical method of avoiding stick-slip motion is to use a lubricant. But even at high normal load and small sliding velocities stick-slip motion occurs [1]. Here we propose an active way to avoid stick-slip motion. It is inspired by the methods used in controlling chaos [2,3]. The idea is not to change the physics at the friction interface but to stabilize unstable states.

In our case the unstable state is the continuous-sliding state. It is unstable for velocity-weakening friction laws where the friction force decreases with increasing sliding velocity [4]. We use the delayed-feedback method proposed by Pyragas for stabilizing periodic orbits in a chaotic attractor [3]. The feedback variable is the elastic deformation (see below). Our parameter of controlling is either the sliding velocity or the normal load.

In order to show that our proposed methods work, we have studied analytically as well as numerically a simplified model for the lateral motion of two solid bodies in contact (see Fig. 1). We assume that one of the bodies is fixed whereas the other one (with mass M) can slide. The elasticity of the sliding bodies (or the whole machinery) is modeled by a spring with stiffness κ . There is a normal load N which presses the bodies against each other and there is the applied velocity v which is generally different from the relative sliding velocity \dot{x} of the bodies. We assume that the lateral degree of freedom x is the most important one. We therefore neglect all other ones. The friction force F at the interface is proportional to N in accordance with Amonton's law, [1]. The equations of motion read

$$M\ddot{x}(t) = \kappa a(t) - \mu_K(\dot{x}(t))N, \quad \text{if } \dot{x}(t) \neq 0, \quad (1a)$$

or

$$\dot{x}(t) = 0, \quad \text{if } |\kappa a(t)| < \mu_S N, \quad (1b)$$

and

$$\dot{a}(t) = v - \dot{x}(t), \quad (1c)$$

where μ_S is static friction coefficient and $\mu_K(x)$ is the kinetic friction coefficient which in general depends on the sliding velocity. The variable a denotes the spring elongation (i.e., the difference between the stage position and the block position). In general it measures the elastic deformation of the bodies due to the contact. Our feedback variable is a . The control parameter is either the applied velocity v or the normal load N . Hence we replace either v by

$$v = v_0 + \alpha_v[a(t) - a(t - \tau)] \quad (2a)$$

or N by

$$N = N_0 + \alpha_N[a(t) - a(t - \tau)], \quad (2b)$$

where v_0 and N_0 are the unperturbed applied velocity and normal load, respectively, τ is the delay time, and α_v and α_N are the amplitudes of control. Note, that load control works only if N is always positive. Otherwise it would lead to a lift-off of the sliding bodies. We will first calculate analytically where in the space of control amplitudes and delay time the continuous-sliding state is stable.

The continuous-sliding state is given by $\dot{x} = v_0$ and $a = \mu_K(v_0)N_0/\kappa$. In order to test its stability we make the

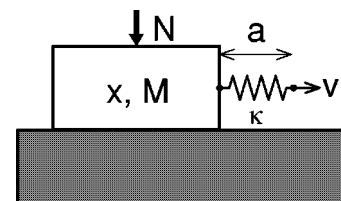


FIG. 1. The stick-slip oscillator. The feedback variable of the proposed controlling methods is the elastic deformation a . The control parameter is either the applied velocity v or the normal load N .

ansatz $\dot{x}(t) = v_0 + c_x \exp(\lambda t)$ and $a(t) = \mu_K(v_0)N_0/\kappa + c_a \exp(\lambda t)$ and linearize the equation of motion in c_x and c_a . Nontrivial solutions are possible only if λ fulfills the following characteristic ‘‘polynomial’’:

$$M\lambda^2 + [\mu'_K(v_0)N_0 - M\alpha_v(1 - e^{-\lambda\tau})]\lambda + \kappa - [\mu'_K(v_0)N_0\alpha_v + \mu_K(v_0)\alpha_N](1 - e^{-\lambda\tau}) = 0. \quad (3)$$

The delay term is responsible for $e^{-\lambda\tau}$ which turns the polynomial into a transcendental equation for λ . Thus Eq. (3) has more than two solutions. In fact, there are infinitely many. Solutions are either real or coming as conjugated complex pairs. The continuous sliding state is stable if the real parts of all solutions of Eq. (3) are negative. It can be shown that the number of solutions with positive real part is finite because the exponential term is bounded. In fact one can give upper limits of the real part and the imaginary part of the solutions. Actually we do not need to know the solutions of Eq. (3). We only want to know the stability boundary in the space of the control amplitudes α_v , α_N , and the delay time τ . The stability boundary is a two-dimensional manifold where a solution of Eq. (3) crosses the imaginary axis. Such manifolds can be calculated analytically in parametric form. They are solutions of

$$2A_r B_r + 2A_i B_i = B_r^2 + B_i^2, \quad (4)$$

$$\cos \omega\tau = 1 - \frac{A_r B_r + A_i B_i}{A_r^2 + A_i^2}, \quad (5a)$$

and

$$\sin \omega\tau = \frac{A_r B_i - A_i B_r}{A_r^2 + A_i^2}, \quad (5b)$$

where

$$A_r \equiv \mu_K(v_0)\alpha_N + \mu'_K(v_0)N_0\alpha_v, \quad A_i \equiv M\omega\alpha_v, \quad (6)$$

and

$$B_r = \kappa - M\omega^2, \quad B_i = \mu'_K(v_0)N_0\omega. \quad (7)$$

The parameter ω is the imaginary part of λ . Equation (4) is a linear inhomogeneous equation for α_v and α_N . By giving one α we can express the other one in terms of ω . From Eq. (5) we get a countable set of solutions for τ . Near the manifold one can expand λ into a Taylor series in order to find out whether the real part of λ increases or decreases when the manifold is crossed. Together with the facts that Eq. (3) has analytic solutions for $\alpha_v = \alpha_N = 0$ and the solutions of Eq. (3) are continuous functions of the parameters we are able to find the stable regions in the parameter space.

Figure 2 shows that unstable continuous sliding states can be stabilized with both types of control. Because of the multiplicity of the solutions of Eq. (5) we also get stability regions for larger values of τ . But these are less interesting because of larger time scales on which the system relaxes into the continuous sliding state. Pure velocity control (i.e.,

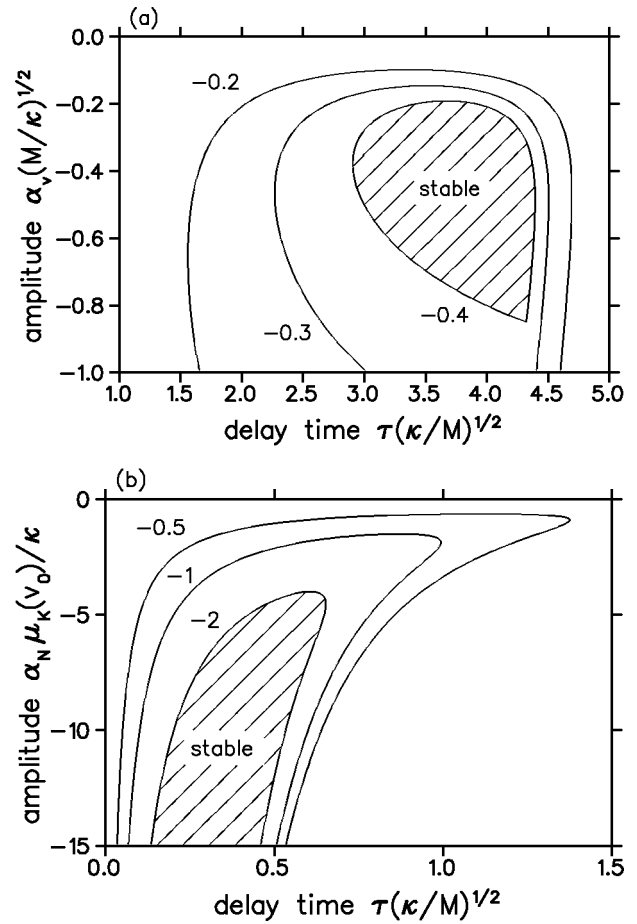


FIG. 2. Boundaries of stable continuous sliding for (a) pure velocity control (i.e., $\alpha_N = 0$) and (b) pure load control (i.e., $\alpha_v = 0$). Different boundaries are denoted by the values of $\mu'_K(v_0)N_0/\sqrt{\kappa M}$.

$\alpha_N = 0$) is less efficient than load control because the stability area is smaller and it disappears if $\mu'_K(v_0)N_0 \lesssim -0.68\sqrt{\kappa M}$.

In pure load control there is always a stable region. Only the minimal value of $|\alpha_N|$ increases with increasing $|\mu'_K|$. For $\tau \rightarrow 0$ a more simple formula for the stability boundary can be given. For small delay times (i.e., $\tau \ll \sqrt{M/\kappa}$) one can approximate $a(t) - a(t - \tau)$ by the time derivative of a . Hence, the equation of motion becomes a differential equation and Eq. (3) becomes a second-order polynomial since $(1 - e^{-\lambda\tau}) \rightarrow \lambda\tau$. It is easy to see that pure velocity control (i.e., $\alpha_N = 0$) is not able to stabilize unstable continuous sliding states whereas for pure load control (i.e., $\alpha_v = 0$) the stabilization works if

$$\alpha_N < \frac{\mu'_K(v_0)N_0}{\mu_K(v_0)\tau}. \quad (8)$$

We have confirmed our analytical results by numerical simulations of the equation of motions. Furthermore, they show that the basin of attraction of a formerly unstable continuous sliding state can be quite large. The examples in Fig. 3 show that stick-slip oscillations disappear after the control is switched on. In our simulations we found that stick-slip motion survives if the applied velocity v_0 is below some

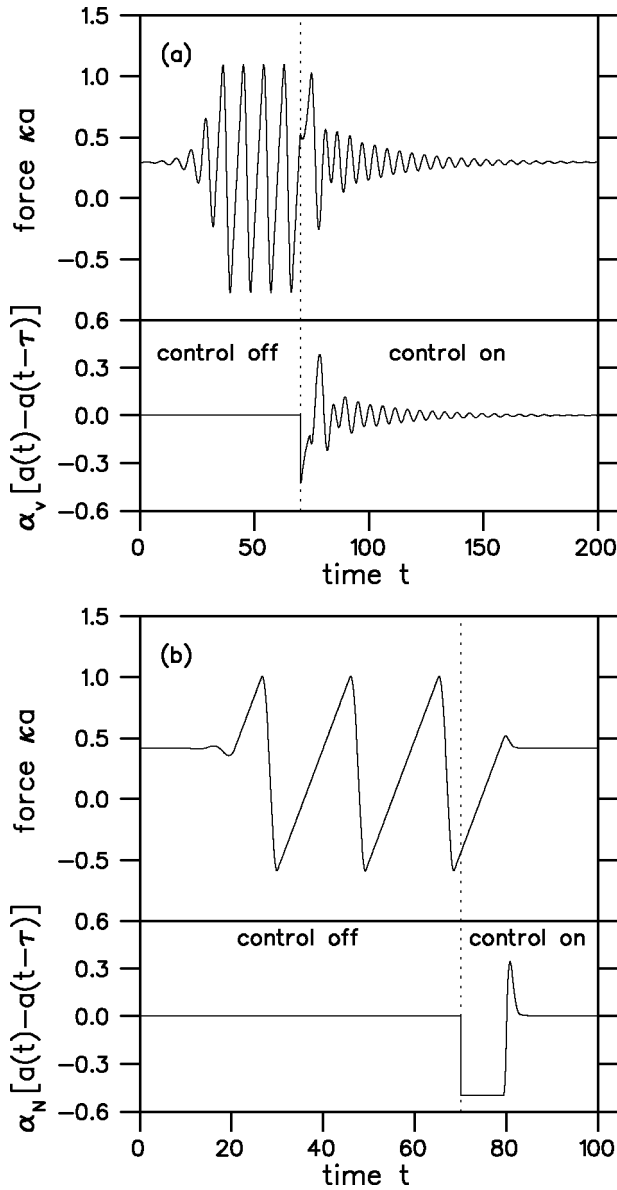


FIG. 3. Turning stick-slip motion into continuous sliding. The simulations of Eq. (1) were done with the purely velocity-weakening friction law $\mu_K(\dot{x}) = \mu_0 / (1 + \dot{x}/v_s)$ with $\mu_0 = 0.5$ and $v_s = 0.5$. The simulation starts with an initial state near the unstable continuous sliding state. The instability leads to stick-slip motion. At $t = 70$ the control was switched on [velocity control in case (a) and load control in case (b)]. The units of t , κa , $\alpha_v[a(t) - a(t - \tau)]$, and $\alpha_N[a(t) - a(t - \tau)]$ are $(M/\kappa)^{1/2}$, $\mu_s N_0$, $\mu_s N_0 / (M\kappa)^{1/2}$, and N_0 , respectively. The parameters are $M = \kappa = \mu_s = N_0 = 1$, (a) $v_0 = 0.35$, $\alpha_v = -0.33$, $\alpha_N = 0$, $\tau = 4.15$, and (b) $v_0 = 0.1$, $\alpha_v = 0$, $\alpha_N = -10$, $\tau = 0.5$.

critical value v_c . Again pure velocity control is less robust than load control. For example, stick-slip motion cannot be destroyed by pure velocity control for that value of v_0 for which in Fig. 3(b) load control easily turns stick-slip motion into continuous sliding. As a rule of thumb we found that in the case of pure velocity control the sticking time has to be of the same order or less than the slipping time.

In the case of load control it is possible to turn stick-slip motion into continuous sliding for arbitrary small values of v_0 . To see this we discuss an analytically treatable case. We

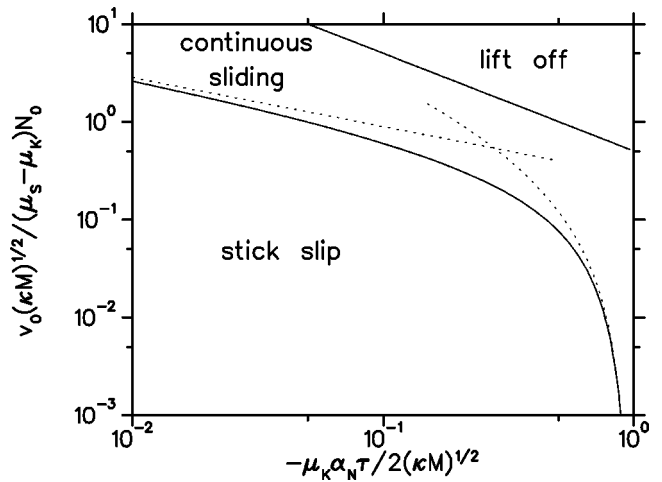


FIG. 4. Boundary of stick-slip motion. The lower solid curve is v_c for $\tau \rightarrow 0$ and Coulomb's law (i.e., $\mu_K = \text{const} < \mu_s$). The dotted curves are analytic approximations (for more details, see the main text). The upper solid curve is the lift-off condition. The ratio of both friction coefficients is $\mu_K / \mu_s = 0.5$.

assume that μ_K does not depend on the sliding velocity (Coulomb's law). Furthermore we restrict ourselves to the limit $\tau \rightarrow 0$ (but $\alpha_N \tau$ finite). Both assumptions turn the equation of motion into a linear differential equation for the slip motion:

$$M \frac{d^2 \dot{x}}{dt^2} - \mu_K \alpha_N \tau \frac{d\dot{x}}{dt} + \kappa(\dot{x} - v_0) = 0, \quad (9)$$

with the initial conditions

$$\dot{x}(0) = 0, \quad \frac{d\dot{x}}{dt}(0) = \frac{\mu_s - \mu_K}{M} (N_0 + \alpha_N \tau v_0), \quad (10)$$

assuming that the slip motion is just starting at $t = 0$. The continuous sliding state is stable for $\alpha_N < 0$. It will be approached in the long-time limit if $\dot{x}(t) > 0$, for $t > 0$ (i.e., no resticking). For $\alpha_N \tau < -2\sqrt{\kappa M} / \mu_K$, Eq. (9) describes an overdamped harmonic oscillator. Thus the solution $\dot{x}(t)$ increases monotonically and never resticks. Therefore stick-slip motion disappears even for infinitesimal small v_0 , i.e., $v_c = 0$. In the underdamped case one gets resticking if $v_0 < v_c$. To calculate v_c one has to solve $\dot{x}(T) = \ddot{x}(T) = 0$, with $T > 0$. This can be done numerically. The result for $\mu_K = 0.5\mu_s$ is shown in Fig. 4. In the limits $\alpha_N \rightarrow 0$ and $\alpha_N \rightarrow -2\sqrt{\kappa M} / \mu_K \tau$ we get the approximations $v_c = \tilde{v} / 2\sqrt{\pi} \gamma$ and $v_c = \tilde{v} \gamma^{-1} \exp(-1 - \pi\gamma / \sqrt{1 - \gamma^2})$, resp., where $\tilde{v} \equiv (\mu_s - \mu_K)N_0 / \sqrt{\kappa M}$ and $\gamma \equiv -\mu_K \alpha_N \tau / 2\sqrt{\kappa M}$. Note that v_0 has to be less than $N_0 / (-\alpha_N \tau)$ otherwise the control mechanism would lead to a lift-off in the sticking phase. It is easy to show that the lift-off curve is always above v_c (see Fig. 4).

Recently Rozman *et al.* proposed a different method for stabilizing the continuous sliding state [6]. Their method is similar to the method of Ott *et al.* for stabilizing periodic orbits in a chaotic attractor [2]. For this method one has to reconstruct the Poincaré return map near the unstable orbit.

This is done by observing the system dynamics without control. Rozman *et al.* used the normal force as the parameter of controlling.

Compared with our method the advantage of the method of Rozman *et al.* is that one does not have to rely on macroscopic equations of motion like Eq. (1). Such equations of motion are reliable for large sliding velocities but it is well known that they may not be correct for small velocities, especially in the case of transitions from sticking to sliding and vice versa [5,7–10]. In fact Rozman *et al.* have tested their method for a simple model where in addition to the macroscopic degree of freedom (i.e., the position of the sliding block) an internal degree of freedom appears which describes the state of a lubricant.

There are two disadvantages of the method of Rozman *et al.* First, controlling methods of Ott *et al.* work only in the vicinity of periodic orbits. If these orbits are embedded in a chaotic attractor the system will eventually come close to them. Therefore, turning stick-slip motion into continuous sliding is possible only, if the stick-slip motion is erratic enough to be close to the continuous sliding state. Otherwise, the method works only if one starts at a large stage velocity where the continuous sliding state is already stable and then slowly decreases the velocity below the value where the continuous sliding state becomes unstable [6]. A delayed-feedback method is not restricted to the vicinity of the unstable orbit. Of course starting far away from the orbit may

lead to strong controlling forces at the beginning (see Fig. 3). But they decay exponentially by approaching the orbit. The second disadvantage of the method of Rozman *et al.* is the necessity of reconstructing the dynamics. This may be more or less difficult depending on the details of the dynamics of the internal degrees of freedom at the friction interface. For technological applications this might be important, especially because the reconstruction has to be recalibrated from time to time.

In this Rapid Communication we have introduced two robust methods of stabilizing continuous sliding. They are also able to destroy regular stick-slip motion. Both methods rely on a delayed feedback where the feedback variable is the elastic deformation of the sliding bodies or the machinery. The controlling parameter is either the applied velocity or the normal load. The velocity control is less robust than the load control.

There are two fields of application of controlling friction. Obviously there will be technological applications for reducing vibration and wear. But controlling friction experiments can also be used to increase our understanding of the physics of dry friction. For example, using these methods one can measure the effective friction force as a function of the sliding velocity even in the velocity-weakening regime.

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